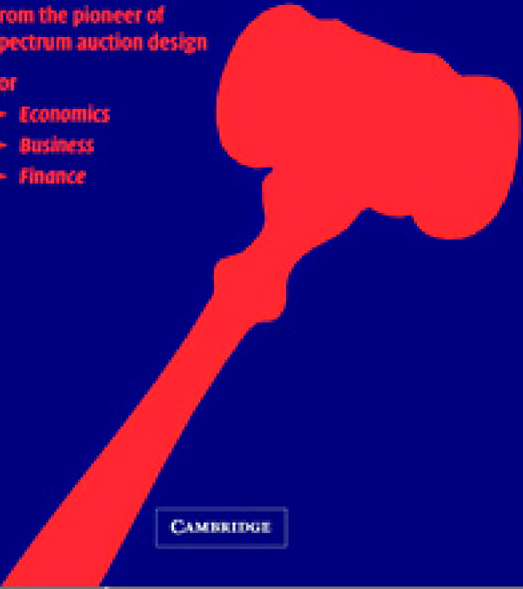
Exhibit I

Paul Milgrom

Putting Auction Theory to Work

From the pioneer of spectrum auction design

For
- Economics
- Business
- Finance



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The use of the envelope theorem in this proof is typical, so it is worthwhile to build intuition by restating the argument in words. Holmstrom's formula (3.10) is the technical part. It establishes a *necessary condition* for how a bidder's cash payments can vary with his type, given the rule z specifying decision outcomes. Together, the decision outcome and the payoff of the lowest type fix a unique payment rule. For the Vickrey auction, the lowest type is a bidder that always loses the auction and has a payoff of zero. Generally, the VCG mechanism corresponding to the function h is the unique mechanism with properties (i) and (ii) in which a losing bidder i pays the amount $h^i(t^{-i})$.

Expressing the participant's maximal payoff as the integral of the partial derivative of the payoff function has long been an important step in optimal mechanism design problems. Mirrlees (1971), Holmstrom (1979), Laffont and Maskin (1980), Myerson (1981), Riley and Samuelson (1981), Fudenberg and Tirole (1991), and Williams (1999) all derived integral conditions in particular models by restricting attention to piecewise continuously differentiable choice rules or even narrower classes. However, it may be optimal to implement a choice rule that is not piecewise continuously differentiable. One example is the class of trading problems with linear utility described in chapter 6.5 of Myerson (1991). The integral form envelope theorem gives us the necessary tool for dealing with the full range of possibilities.

We next see that very much the same argument can be applied in the context of Bayesian equilibrium. As in the dominant strategies application, the formula sharply limits the payment rules that can apply at equilibrium.

3.3.3 Myerson's Lemma[6]

In practice, many designers, regulators, and observers of auctions have falsely high expectations about how changes in the rules can affect prices and payoffs. Many believe auction procedures can affect expected selling prices and bidders' payoffs without affecting the way the goods are allocated.

[6] Most expositions of incentive theory treat payoff equivalence and revenue equivalence as a single result, but that seems to me a mistake. That treatment not only obfuscates the close connections between incentive theory and demand theory, it also impedes applications to models with risk averse decision makers or in which outcomes are inefficient. The approach taken here makes it straightforward to treat these additional developments.

According to current economic theory, an auction designer's ability to manipulate prices and payoffs without changing allocations is much more limited. Here, we examine what the auction design can do when bidders play Bayes–Nash equilibrium strategies, bidding optimally given their beliefs about others' types and strategies.

Definition. A strategy profile σ is a *Bayes–Nash equilibrium* of the mechanism $\Gamma = (S, \omega)$ in environment $(\Omega, N, [0,1]^N, u, \pi)$ if for all t^i,[7]

$$\begin{aligned}\sigma^i(t^i) &\in \arg\max_{\tilde{\sigma}^i \in S^i} E^i[u^i(\omega(\tilde{\sigma}^i, \sigma^{-i}(t^{-i})), \vec{t}) | t^i] \\ &= \arg\max_{\tilde{\sigma}^i \in S^i} \int u^i(\omega(\tilde{\sigma}^i, \sigma^{-i}(t^{-i})), \vec{t})\, d\pi^i(t^{-i} | t^i). \end{aligned} \tag{3.14}$$

In most of this chapter, we study a *standard independent private values* model. This entails the assumptions that

(i) the types are $\Theta^i = [0, 1]$,
(ii) payoffs are quasi-linear, as described above, and bidders are risk neutral,
(iii) values are *private* ($v^i(x, \vec{t}) \equiv v^i(x, t^i)$),
(iv) types are statistically independent, and
(v) the conditions of the integral form envelope theorem (theorem 3.1) are satisfied.

With these assumptions, expected payoffs can be written as follows:

$$\begin{aligned}&E^i[u^i(\omega(\tilde{\sigma}^i, \sigma^{-i}(t^{-i})), \vec{t}) | t^i] \\ &\quad = E^i[z(\tilde{\sigma}^i, \sigma^{-i}(t^{-i})) \cdot v^i(t^i) - p^i(\tilde{\sigma}^i, \sigma^{-i}(t^{-i}))]. \end{aligned} \tag{3.15}$$

Let $V^i(t^i)$ denote the maximum expected payoff of player i of type t^i in the game. Then

$$V^i(t^i) = \max_{\tilde{\sigma}^i} E^i[z(\tilde{\sigma}^i, \sigma^{-i}(t^{-i})) \cdot v^i(t^i) - p^i(\tilde{\sigma}^i, \sigma^{-i}(t^{-i}))]. \tag{3.16}$$

In close analogy to Holmstrom's lemma, we have the following:

Theorem 3.3 (Myerson's lemma; payoff equivalence theorem). Consider a standard independent private values model, and suppose that σ is a Bayes–Nash equilibrium of the game corresponding to

[7] In this expression E^i refers to an expectation computed with respect to the beliefs of player i.

$(\Omega, N, S, \omega, [0,1]^N, v, \pi)$ with full performance (x, p). Then the expected payoffs satisfy

$$V^i(\tau) = V^i(0) + \int_0^\tau E^i[z(\vec{t})|t^i = s] \cdot \frac{dv^i}{ds}\, ds. \tag{3.17}$$

In particular, if V^i is differentiable at τ, then $\frac{\partial}{\partial\tau} V^i(\tau) = E^i[z(\vec{t})|t^i = \tau] \cdot dv^i(\tau)/d\tau$. Expected payments must satisfy

$$\begin{aligned} E^i[p^i(\vec{t})|t^i = \tau] &= -V^i(0) + E^i[z(\vec{t})|t^i = \tau] \cdot v^i(\tau) \\ &\quad - \int_0^\tau E^i[z(\vec{t})|t^i = \tau] \cdot \frac{dv^i}{ds}\, ds. \end{aligned} \tag{3.18}$$

Proof. Equation (3.17) follows directly from (3.16) and the envelope theorem. The derivative form of the theorem follows by differentiating (3.17) with respect to τ. At equilibrium, a player's expected payoff is $V^i(\tau) = E[z(\vec{t})|t^i = \tau] \cdot v^i(\tau) - E[p^i(\vec{t})|t^i = \tau]$. Substituting that into (3.17) and rearranging yields (3.18). ∎

If we compare two different auction mechanisms in which the lowest types of bidders always lose and pay zero, then $V^i(0) = 0$ for both. If the outcome function z is also the same for both, then according to the theorem, bidders' expected payoffs and payments are also the same. Provided our model of strategic bidders is right, this conclusion contradicts intuitive claims that one can change bidder payoffs by manipulating rules without reducing efficiency.

3.3.4 Revenue Equivalence Theorems

The (risk neutral) payoff equivalence theorem applies to bidder payoffs, but it also has immediate implications for the seller's expected revenues. The original theorem of this sort is Myerson's revenue equivalence theorem, which applies to auctions of a single good. We begin with a recent extension reported by Williams (1999).

As above, $(\hat{x}, \hat{p})$ denotes the VCG pivot mechanism.

Theorem 3.4. Consider a standard independent private values model, and suppose that σ is a Bayes–Nash equilibrium of the game corresponding to $(\Omega, N, S, \omega, [0,1]^N, v, \pi)$ with full performance $(\hat{x}, p)$. Then the expected payment to the mechanism operator is the same as for the VCG mechanism $(\hat{x}, \hat{p} + h)$, where $h^i(t^{-i}) \equiv E^i[p^i(0, t^{-i})]$.

Proof. Because the always optimal equilibrium of the VCG mechanism is also a Bayes–Nash equilibrium, Myerson's lemma applies to it, with $p = \hat{p} + h$ and $V^i(0) = 0$. So the expected total revenue is $E[\sum_{i\in N} p^i(\vec{t})] = E[\sum_{i\in N} E[p^i(\vec{t})|t^i]] = E[\sum_{i\in N} E[\hat{p}^i(\vec{t})|t^i]] = E[\sum_{i\in N} \hat{p}^i(\vec{t})]$. ■

The famous revenue equivalence theorem of auction theory is a special case:

Corollary. Consider a standard independent private values model with a single indivisible good for sale and in which losers' payoffs are zero. Suppose that σ is a Bayes–Nash equilibrium of the corresponding game $(\Omega, N, S, \omega, [0, 1]^N, v, \pi)$. Suppose the full performance is $(\hat{x}, p)$. Let $v^{(1)}$, $v^{(2)}$, ... denote the order statistics of the bidder valuations for the single good, from highest to smallest. Then the total expected payment by participants in the mechanism is $E[v^{(2)}]$.

Proof. Observe that $v^{(2)}$ is the sales revenue associated with the Vickrey mechanism in this environment, and apply theorem 3.4. ■

The preceding version of the revenue equivalence theorem is the best-known theorem in auction theory. The history of the theorem begins with Vickrey, who computed equilibria for four different auction mechanisms and made the then surprising discovery that the expected revenues were exactly the same in each of them. Simultaneous contributions by Myerson (1981) and by Riley and Samuelson (1981) implicitly established the reason in terms of the envelope and payoff equivalence theorems, as described above.

Various extensions of the standard revenue equivalence theorem are possible by adapting the same argument to more general models. The following one is a version that applies to the interdependent values model of Milgrom and Weber (1982), provided the types are statistically independent.

Theorem 3.5. Consider a standard independent private values model with a single indivisible good for sale in which losers' payoffs are zero and the private values condition is replaced by the condition that each bidder i's value for the good satisfies $v^i = v(t^i, t^{-i})$, where v is continuously differentiable. Suppose that σ is a Bayes–Nash equilibrium of

the corresponding game $(\Omega, N, S, \omega, [0,1]^N, v, \pi)$. If the bidder with the highest type always wins the auction, then the expected payoff of each bidder i is $E[\int_0^{t^i} v_1(s, t^{-i})\, ds]$ and the seller's expected revenue is

$$E\left[v(t^{(1)}, t^{-(1)})\right] - N \cdot E\left[\int_0^{t^i} v_1(s, t^{-i})\, ds\right].$$

Proof. The bidder's payoffs are the ones determined in the now-familiar way from the envelope theorem formula. The total expected payoff is $E[v(t^{(1)}, t^{-(1)})]$, so the seller's expected payoff is the total minus the sum of the bidders' expected payoffs. ■

One important use of the revenue equivalence theorems is as a benchmark for analyzing cases when the assumptions of the theorems do not hold. In the next chapter, we will see how budget constraints, risk aversion, endogenous quantities, and correlation of types all lead to systematic predictions comparing expected revenues from different kinds of auctions, even ones with the same decision performance. Of course, mechanisms with different decision performance will also have different levels of expected revenue. This is potentially important because standard auctions in asymmetric environments generally have different decision performance.

3.3.5 The Myerson–Satterthwaite Theorem

Another famous early problem of mechanism design theory is designing efficient exchange between a buyer and a seller when both have uncertain types. These situations are often known as the bilateral monopoly or bilateral trade problem. Earlier developments in transaction cost economics and bargaining theory had treated it as an axiom that exchange will take place whenever that is necessary for efficiency. This *efficiency axiom* is explicit in the derivations of the Nash bargaining solution, the Kalai–Smorodinsky solution, and the Shapley value, as well as in many treatments of the so-called Coase theorem.

Doubts about the efficiency axiom are based partly in concerns about bargaining with incomplete information. After all, a seller is naturally inclined to exaggerate the cost of his good, and a buyer is inclined to pretend that her value is low. Should we not expect these exaggerations to lead sometimes to missed trading opportunities? Is the problem of exaggeration in bargaining a fundamental one? Or can a bargaining